What did we learn from studying acoustic black holes ?

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Abstract

The study of acoustic black holes has been undertaken to provide new insights about the role of high frequencies in black hole evaporation. Because of the infinite gravitational redshift from the event horizon, Hawking quanta emerge from configurations which possessed ultra high (trans-Planckian) frequencies. Therefore Hawking radiation cannot be derived within the framework of a low energy effective theory; and in all derivations there are some assumptions concerning Planck scale physics. The analogy with condensed matter physics was thus introduced to see if the asymptotic properties of the Hawking phonons emitted by an acoustic black hole, namely stationarity and thermality, are sensitive to the high frequency physics which stems from the granular character of matter and which is governed by a non-linear dispersion relation. In 1995 Unruh showed that they are not sensitive in this respect, in spite of the fact that phonon propagation near the (acoustic) horizon drastically differs from that of photons. In 2000 the same analogy was used to establish the robustness of the spectrum of primordial density fluctuations in inflationary models. This analogy is currently stimulating research for experimenting Hawking radiation. Finally it could also be a useful guide for going beyond the semi-classical description of black hole evaporation.

The motivation for the analogy

The semi-classical description of black hole evaporation is based on the hypothesis that the dynamics of the system (black hole + radiation) can be well approximated by treating the gravitational field classically. In this case the quantized radiation field ϕ obeys the d'Alembert equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\phi = 0\,, \tag{1}$$

where $g^{\mu\nu}$ is the metric of the collapsing body. Following Hawking[1], one obtains that an isolated black hole formed by gravitational collapse radiates

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away its mass by emitting a thermal flux with a temperature given by $T = \kappa/2\pi$ where κ is its surface gravity.

The hypothesis of having treated $g^{\mu\nu}$ classically finds support in the fact that $\langle T_{\mu\nu} \rangle$, the expectation value of the energy-momentum tensor of ϕ, is regular. By solving Einstein equations driven by $\langle T_{\mu\nu} \rangle$, one then finds that the evaporation stays adiabatic as long as the residual mass of the hole is much larger than the Planck mass, see e.g. [2].

However, the validity of the semi-classical description becomes dubious when inquiring about the origin of Hawking quanta. Eq. (1) predicts that they emerge from configurations localized extremely close to the horizon and characterized by ultra-high frequencies (when measured by an infalling observer). This is specific and intrinsic to horizon quantum physics as it follows from free propagation near an event horizon and from second quantization. The infinite gravitational redshift from the event horizon implies that during the last stage of the collapse the radiation emitted by the body is redshifted according to

$$\lambda(t) \propto \Omega \ e^{-\kappa t}, \tag{2}$$

where Ω is the emitted frequency and λ that received on $\mathcal{J}^+$. Classically, there is thus no appeal to high frequencies since the initial frequencies Ω are bounded. On the contrary, Hawking quanta emerge from vacuum fluctuations with very high Ω precisely because these are redshifted as in Eq. (2). Indeed, the steady character of Hawking radiation is achieved at time t by referring to vacuum frequencies which grow according to $\Omega(t) \propto \kappa e^{\kappa t}$, see Eq. (3.34) in [2].

Hawking radiation therefore cannot be derived within the framework of a low energy effective theory. Moreover since Planck scale physics is unknown, it is important to determine if/why the asymptotic properties of Hawking radiation could be insensitive to it. From the outset[3] this question has been the principle motivation for studying acoustic black holes.

The acoustic black hole geometry

The concept of acoustic black hole arises from the analogy between light propagation in a curved background and sound propagation in a moving fluid[3, 4]. Consider a vorticity-free flow of a barotropic fluid. The velocity field can be written as $v = \nabla\Psi$ and the pressure P is related to the density ρ by an equation of state $P = P(\rho)$. Consider also the equation of continuity and Euler equation. Then linearize these three equations about a background solution, $v_B = \nabla\Psi_B$, P_B, ρ_B, and consider the equation for $\psi = \Psi - \Psi_B$, see [4] for details. Verify that ψ obeys Eq. (1) when introducing the 4D acoustic metric $g_{\mu\nu}^{ac}$ given (up to a conformal factor) by $g_{tt}^{ac} = -(c_s^2 - v_B^2)$, $g_{ti} = -v_{B\,i}$, $g_{ij} = \delta_{ij}$. The velocity of sound c_s is given by $c_s^2 = dP/d\rho$.

To obtain an acoustic geometry which behaves like that of a black hole, one must consider a flow which becomes supersonic without developing any shock wave. A well-known device (by hydrodynamicians) which realizes this is the Laval nozzle: a pipe wherein $|v_B|$ reaches c_s at its narrowest radius.

When considering a flow to the left in the x-direction, the velocity behaves as $v_B(x) = -c_s + x\kappa_{ac}$ and the acoustic horizon is located at $x = 0$. In the near horizon region, for $x \ll c_s/\kappa_{ac}$, null (retarded) geodesics behave like

$$x(t) = x_0\, e^{\kappa_{ac}(t-t_0)}\ . \tag{3}$$

Hence, for $x < 0$, right moving sound waves are carried away to the left by the supersonic flow. For $x > 0$ instead, they propagate to the right and their frequencies are redshifted according to Eq. (2) with $\kappa = \kappa_{ac}$.

Thus, if the phonon field ψ is second quantized and if its state is regular across the horizon, the standard treatment[1, 2] applies. It tells us that a constant flux of phonons at temperature $T_{ac} = \kappa_{ac}/2\pi$ will be emitted against the flow, to the right. The quantization of the phonon field is physically pertinent in He^4 and in Bose-Einstein condensates (BEC). The regularity of the state of ψ requires that the flow $v_B(x)$ be regular across the horizon.

The usefulness of the analogy

So far we have not learned much because we have not yet taken into account the fact that the dispersion relation of phonons is not linear when the wave length approaches the inter-atomic distance, d_c. Three possibilities can be obtained. Two can be modeled by

$$\Omega^2 = c_s^2 k^2 [1 + \xi\, k^2 d_c^2 + O(k^4)]\,, \tag{4}$$

with $\xi = \pm 1$. $\xi = 1$ is realized in BEC and leads to supersonic propagation. $\xi = -1$ is realized in He^4 and leads to subsonic propagation. The third case describes propagation in an absorptive medium such as a viscous fluid[4]. It is governed by a correction term, the bracket in Eq. (4), given by $[1 - i|k|d_c]$.

When taking into account the non-linearities of Eq. (4) while deriving the production of Hawking phonons, one can *test* if the asymptotic properties of the flux are sensitive to these non-linearities. In 1995 Unruh[3] showed (numerically and for $\xi = -1$) that they are not sensitive when $T_{ac} \ll c_s/d_c$ and when the initial state is vacuum for $\Omega \gg T_{ac}$. These conclusions are now understood analytically, see [5] for a review. In spite of this absence of asymptotic modification, when propagating backwards in time a mode (or better a wave packet), one finds that its near horizon propagation is dramatically modified when its blue-shifted frequency reaches the critical frequency $\Omega_c = c_s/d_c$, see [6] for schematic representations of the modified trajectories.

Being insensitive to high energy physics, the asymptotic Hawking radiation is truly a low energy $O(T)$ phenomenon. This reinforces our confidence in the low energy predictions of the semi-classical treatment but does not take us any further concerning its dubious high energy predictions.

Inflationary models

In inflationary scenarios, the primordial density fluctuations find their origin in the non-adiabatic evolution of the non-homogeneous modes of the inflaton which were in their vacuum state at the onset of inflation. Given the exponentially growing redshift during inflation, their initial frequency Ω was proportional to $e^{H\Delta t}$, where H is the Hubble constant and $H\Delta t$ is the number of e-folds during inflation. If the latter is taken to be larger than ~ 70 (which is the minimal number for inflation to work when the adiabatic era starts just below GUT scale, near 10^{15} Gev) there is an appeal to Planckian physics. Therefore the doubts concerning Hawking radiation apply here as well[5]. Thus the appeal to modified dispersion relations can also be called upon to see if the scale-invariance of the primordial fluctuations spectrum is affected by modifications of Planck scale physics[7].

As in black hole physics, the properties of the spectrum are robust. The two conditions guaranteeing this robustness are the adiabaticity of the evolution from Planck scale to that of H, and the regularity of the initial configurations. The adiabaticity essentially follows from scale separation, $M_{Planck} \gg H$, whereas the second condition is severely constrained by self-consistency in the following sense[8]. The very existence of inflation implies that the dominant term in the Friedman equation is the inflaton potential energy. Thus the contribution of the other modes must be negligible. When $H \ll M_{Planck}$ and when inflation lasts more than 70 e-folds, this condition requires that the initial state of the modes observed today be vacuum.

Experiments ?

Can we conceive and realize experiments in which Hawking quasi-particles could be measured ? We used the term "quasi-particles" rather than phonons to emphasize that in condensed matter there is a large variety of collective excitations which can propagate in effective metrics. At present it is not clear which material is the most suitable for our purpose. Another challenge is to build a regular horizon enclosing a trapped region. But probably the main difficulty is to get a sufficiently high temperature. Indeed it does not seem easy to get $T_{ac} > 10^{-6}$K.

To overcome the detection of these soft quanta, one should perhaps consider a pair of acoustic holes. A natural feature of acoustic holes is to come in pairs: it suffices to reduce the velocity flow v_B so as to re-cross the sound velocity. Moreover this second crossing leads to a white hole. And white holes are unstable. This instability would show up differently according the sign of ξ. For positive ξ, one would obtain a self-amplifying flux of soft phonons, a laser effect, see [9]. On the other hand, for ξ negative, the "partners" of Hawking quanta would leak out of the white horizon with a frequency close to the cutoff $\Omega_c = c_s/d_c$ and would thus be easier to detect.

Orienting research on Quantum Gravity ?

Even though this is clearly a delicate enterprise and one which might turn out to be misleading, it has been one of the major reasons to study acoustic holes. Let us take some risk and present our opinion as it stands today.

Besides the insensitivity of asymptotic properties, we have also learned that for *all* dispersion relations but the scale invariant one $d_c = 0$, there is no appeal to unbounded frequencies. [When the gradient dv_B/dx vanishes for $x - x_{horizon} \gg d_c$, the frequency of a backwards in time propagated wave packet stays of the order of Ω_c for both $\xi = \pm 1$, without any further blueshift. One learns from this that the stationarity of Hawking radiation now results, as in the Golden Rule, by a steady increase of the density of modes at some fixed frequency. This is not what is found when there is no dispersion, see Eq. (3.34) in [2].] This suggests that the quantum gravitational interactions neglected in the semi-classical treatment might introduce a **new** scale d_{QG} which could act on light propagation near the event horizon as d_c did on phonons. In order words, instead of questioning the robustness of asymptotic properties, we are lead to question **is $\Omega = k$ robust** ?

I would claim probably not. Because $\Omega = k$ follows from Eq. (1) which results from a linearisation procedure: when starting from a quantum formulation of black hole evaporation, Eq. (1) is obtained by discarding all fluctuations of $g_{\mu\nu}$ around the mean metric, a solution of the semi-classical Einstein equations. A first order analysis shows that the effects of metric fluctuations on configurations giving rise to Hawking quanta grow like $1/(r - 2M)$. Therefore a taming mechanism is bound to occur. This has been substantiated in [10] where it is shown that vacuum energy fluctuations engender metric fluctuations whose random properties induce a damping similar to that obtained by putting $\xi = -i$ in Eq. (4). That near horizon quantum gravitational interactions should induce dissipative effects was anticipated in [2].

Acknowledgements

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